A Mathedology and an Experimental Didactic Experimentation

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ABSTRACT

In this work, an experimental method for a better understanding of the internal ballistics and the influence of the combustion pressure in the linear burn rate is shown.

It is presented as an internal ballistic mathematical model and the simplifying hypotheses. An experimental methodology, using an end-burning motor, is explained. It is presented through 13 (statistic) tests with sucrose (NaNO₃) at pressures between 0.3 and 20 MPa, and calculating the burn rate constant. The development of a motor for propulsion of an experimental rocket, its performance during flight and in statistic tests, is shown. Although this method has didactic purposes it could evaluate reasonably well the sucrose NaNO₃ internal ballistic behavior.

INTRODUCTION

O

This present work is the development of an activity that is taken with engineering and high school students that has the objective to make them apply their knowledge in the project, including construction, tests, and launching of experimental rockets. This group works in Brazil and soon there will be an article about our activities.

After some experiences with zinc-sulfur rockets, it was decided to design a rocket motor that would be predictable and readily available. It was decided for the NaNO₃ (sugar propellant) for its safe manipulation and no toxic problems, A hibliographic research was done to find the linear burn rate equation and it was not found. The references (Vyverman, 79) and (Marchi, 85) suggests an equation for a KNO₃, and (Parkin, 59) gives a velocity for a certain pressure for the same propellant.

As this equation is necessary for the rocket motor project, it was decided to conduct experiments to learn the equation parameter values.

In this work is shown a methodology to obtain the linear burn rate equation of the sucrose NaNO₃, the project, construction, test and use in launchings of a motor tube with this propellant.

MODEL

There is a relationship between the linear burn rate and the motor chamber pressure:

$$r = a \cdot p_e^n$$
 EQ(1)

where:

r = linear burnrate

p_c = motor chamber pressure

a = burnrate constant

n = bumrate pressure exponent

In a motor chamber in equilibrium, it can be written:

$$p_{c} = m_{p} \cdot X (\gamma \cdot R/M \cdot T_{c})^{1/2}$$

$$\Gamma \cdot A$$

$$EQ(2)$$

where:

m_p= propellant mass consumption

y = ratio of specific heats

T_c = Combustion temperature

R = universal gas constant

M = molecular weight of gas products

 $\Gamma = \gamma(2/\gamma + 1)^{(+1)/2(-1)}$

A_t = throat area

X = Mass fraction of gases in exhaust

If the throat area, the propellant mass consumption rate, and the thermodynamic properties are known in the function of the chamber pressure, it is possible to estimate the chamber pressure through a reiterative process.

TERMODYNAMIC PROPERTIES

A computer program was used to calculate the theoretic exhaust gases properties with no chemical reaction through expansion (Gordon, 76). The following properties were calculated for 5, 10, 40, 100, 150, 200 atm:

Table 1 - Thermodynamic Exhaust Gases Properties for Different Pressures

Pressure	Te	γ	X	М	Г	Cf
(atm)	(K)					
-5	1550	1.1791	0.8402	27.83	0.7004	1,089
10	1615	1.1712	0.8145	27.62	0.6998	1,270
20	1677	1.1639	0.7894	27.42	0.6924	1,416
40.	1732	1.1578	0.7672	27.30	0.6882	1,539
100	1785	1.1519	0.7434	26.97	0.6858	1,675
150	1802	1.1501	0.7362	26.90	0.6849	1,728
200	1811	1.1491	0.7321	26.85	0.6844	1,762

Obs.: Cf = Thrust Coefficient

EQUIPMENT

Motor Tube (casing) - In order to measure different burn rates at different pressures a cylindrical tube was used with an end-burning grain. We used a ST-52 tube with the following characteristics:

> Internal diameter: 51mm External diameter: 70mm Chamber length: 300mm

The tube had a forward closure and an end closure that could hold different nozzles:

Table 2 - Dimensions of the Nozzles Used

Throat diameter (mm)	Throat area (mm²)	Exhaust area (mm²)	Expansion rate
4.2	17.64	58.08	3.25
3.2	10.24	23,76	2.32
2.2	4.84	78.54	16.23
1.9	3.61	33.18	9.19
1.7	2.89	33.18	11.47
1.5	2.25	33.18	14.74

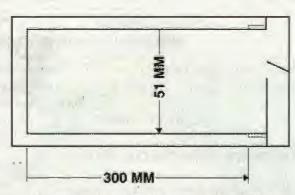


Figure 1 - Motor Tube Used in the Tests

Dynamometer - Some tests were done with the tube in the vertical position and some in the horizontal position in a 4 bar dynamometer.

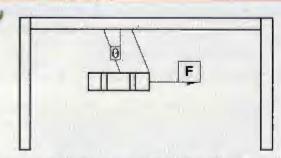


Figure 2 - Motor Tube in the 4 Bar Dynamometer

PREPARATION

For the tests we used end-burning grains.



Figure 3 - Motor Tube with the End-Burning Grain

The inhibition on the walls and in the inner part of the forward closure was accomplished using melted icing sagar. This was applied to the inside of the tube in a thin (3 mm) layer which was formed by pressing a wood rod (dowel) inside the tube.

The propellant grain is composed of 60% NaNO₃ and 40% sucrose. We used the fertilizer form of NaNO₃ because of the price and accessibility. After being weighed, the NaNO₃ is ground in an electric grinder (spinning blade). After this, the NaNO₃ and sucrose are mixed in a glass pot by manually shaking for one to two minutes. The mixture is then heated in a pan within an oil bath. The sucrose is melded, and when the mixture turns a light brown color it is put inside the inhibited motor tube.

Before the grain begins to solidify, 0.5 grams of black powder is put and pressed over the exposed surface of the grain in order to make the ignition easier.

After the grain is solidified its length is measured. The mass of the grain is calculated, assuming a constant density of 1.81 grams/cm³.

THE BURNING OF THE GRAIN

The ignition is done remotely by an electric match and 0.5 grams of black powder. All the tests were recorded by a video

camera. In all the tests a rapid pressurization of the chamber was observed. The burning time was taken by late analysis of the videotape.

BURN RATE AND CHAMBER PRESSURE EVALUATIONS

Burn rate and chamber pressure evaluations were done through 13 statistic tests. A motor tube was used as previously described.

Table 3 - Burn Rate Experimental Data and Results

T	P	Dt1 mm	Dt2	D,	T,	L	r cm/s	P. 10° Pa
1	Н	4.2	2.7	2.7	58	29	0.05	3.30
2	V	4.2	3.9	3.9	25	34	0.13	3,16
3	٧	3.2	2.8	2.8	10	28	0.28	12.84
4	H	2.2	2.0	2.0	8.0	39	0.49	43,60
5	V	2.2	2.0	2.0	7.0	30	0.43	38.27
6	H	1.5	1.3	1.3	2.7	28	1.05	216
7	H	1.5	1.3	1.3	6.7	73	1.08	229
8	V	1.5	1.8	1.65	5.0	141	3.13	383
9	V	1.5	1.3	1.3	-	72	_	-
10	V	1.5	1.3	1.3		69	-	-
11	H	1.7	1.4	1.4	8.5	74	0.88	158
12	٧	1.7	2.2	1.95	7.3	83	1.13	107
13	V	1.9	1.5	1.5	4.8	. 57	1.18	157

OBS: T = tests, V = vertical position of the tube, H = horizontal position of the tube, Dt1 = throat diameter, Dt2 = throat diameter after the test, D_t = admitted throat diameter, T_b = burn time, L = length of the grain, r = burnrate, p_c = chamber pressure.

The burn time was taken with the videotape. The burn rates were admitted constant and were calculated by dividing the grain length by the burn time. The propellant mass consumption rate was calculated by dividing the propellant mass by the burn time. The constant burn rate hypothesis was verified by an almost constant angle of the bars of the dynamometer during the burn time.

In some tests the diameter of the throats were a little smaller by the deposit of NaOH during the test. In order to calculate the chamber pressures the smallest were taken. In the two tests where the throat croded, a medium value was taken.

The chamber pressure p_e was calculated reiteratively by EQ(4) with interpolated values of the thermodynamic properties (see Table 1).

The rear closure was disconnected [from] the tube during tests 9 and 10. Test 8 showed a different behavior from the others. This could have happened by and addition of a 10 mm length of the grain after the first cast. The grain could have burned in the end, and in this interface resulting an expected high pressure. There was difficulty determining the burn time in tests 1 and 11 because in test 1 there was a slow pressurization of 30 seconds and a remaining burn of 28 seconds; so we adopted a burn time of 58 (combined) seconds. In test 11 there was a high exhaust during the first 8.5 seconds with a low exhaust in the last 90 seconds. There was crosion on the nozzles of tests 8 and 12, probably caused by low quality steel.

THE BURN RATE EQUATION

It was calculated an exponential correlation curve by the least squares method. The data of tests 1, 8, 11, and 12 were not used in the calculation of the curve. There was a good correlation of the curve with a good explication coefficient (R2=0.9844).

Follows the burn rate equation.

EQ(3)

where:

$$r = linear burn rate$$
 (cm/s)
 $p_c = combustion chamber pressure (10^5 Pa)$

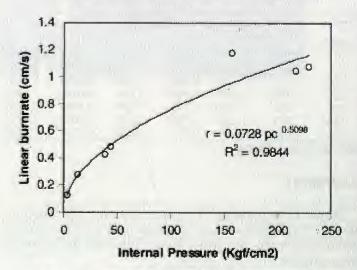


Figure 4 - Burn Rate Curve With Experimental Data

PROJECT OF MOTOR M-1

With the bun rate equation and the thermodynamic data of the exhaust gases, a motor tube project was done for an experimental rocker with unrestricted grain.

Initial parameters:

- P_c=10.10⁵ Pa
- Thrust, F = 100 N
- Burn time = 3.5 s

THROAT AREA CALCULATION

$$A_t = F/C_1 \cdot P_c \qquad EQ(4)$$

where:

Using $P_c = 10$. 10^5 Pa and $C_f = 1.270$ (value obtained in Table 1 for 10 atm) and F = 100 N it is obtained by EQ(4)

$$A_t = 7.87 \cdot 10^5 \text{m}^2$$

BURN SURFACE AREA CALCULATION

By EQ(2) and the above values it is obtained:

but

$$A_b = m_p / r.p$$

r= propellant density A_b= burn area

Using EQ(3) to calculation and the 1.81 g/cm⁵ value for the propellant density, it is gotten:

r = 0.23 cm/s $A_b = 208 \cdot 10^{-4} \text{m}^2$

With these data a motor tube was constructed with the following characteristics:

Burning Chamber Internal Diameter	54.6 mm
Burning Chamber External Diameter	59.2 mm
Throat Diameter	10.0 mm
Exhaust Nozzle Diameter	20.0 mm
Propellant Central Hole Diameter	43.0 mm
Propellant External Diameter	10.0 mm
Propellant Grain Length	125.0 mm

M-1 MOTOR TUBE TESTS

Three launches and one static test were done with the M-I motor. The static test was done in the vertical position and the burn time as taken by videotape analysis. During this test the motor compressed a spring and the burn time was the time the spring was compressed.

We used an experimental rocket with a recovery system for the launches. The thrust force was estimated using the acceleration during the launch pad phase and rocket weight. The burn time was taken using the videotape analysis. It was noticed that the burn rate was higher than the one used in the project probably because of crosive burn effect.

Table 4 - Motor Tube Test Data and Results

Tests	Kind of tests	Burn time (s)	Thrust (N)	apogee	r (cm/s)
1	static	2.7	-	-	0.31
2	flight	2.8	1-		0.30
3	flight	2.7	130	450	0.31
4	flight	3.0	130	450	0.28

Table 5 - Experimental Rocket Data

Rocket mass	1.515 Kg
Motor mass	0.750 Kg
Propellant mass	0.320 Kg
Diameter	63 mm
Length	1.750 m

CONCLUSION

A simple experimental didactic methodology was planned and executed to evaluate and comprehend the propellant burn masteristics.

This methodology proved to be useful to help understand the internal ballistic phenomena, and in the projects construction, test, and launching of experimental rockets.



M1 motor with its propellant grain and mold.

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